Testing for nonlinearity in irregular fluctuations with long-term trends

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We describe a method for investigating nonlinearity in irregular fluctuations (short-term variability) of time series even if the data exhibit long-term trends (periodicities). Such situations are theoretically incompatible with the assumption of previously proposed methods. The null hypothesis addressed by our algorithm is that irregular fluctuations are generated by a stationary linear system. The method is demonstrated for numerical data generated by known systems and applied to several actual time series.

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I. INTRODUCTION

To investigate nonlinearity in irregular fluctuations, various surrogate data methods have been proposed: the Fourier transform (FT), the amplitude adjusted Fourier transform (AAFT), and the iterative AAFT (IAAFT) algorithms [1,2]. Each of these methods is now widely used. All of these techniques are *linear surrogate methods* [3,4], because they are based on a linear process and address a linear null hypothesis. These methods assume stationarity of the data under consideration (that is, data with no trend) as in Fig. 1(a). However, time series exhibiting irregular fluctuations and long-term trends (periodicities) like that shown in Figs. 1(b)–1(d) abound in the real world. Unfortunately, such nonstationary data are theoretically incompatible with the assumption of linear surrogate tests, and the nonstationarity is therefore very likely to lead to incorrect results [1,2]. Dealing with such nonstationary data is difficult.

To investigate irregular fluctuations with long-term trends, a common approach is to separate the irregular fluctuations and long-term trends or to split the time series into segments that can be considered nearly stationary [5]. However, such filtering is not always welcomed because the processed data can lead to spurious results [6]. Hence, it will be preferable, if possible, to investigate features of irregular fluctuations without such pre-processing. Until recently, no surrogate method has been able to tackle such data. Nakamura and Small have proposed the small shuffle surrogate (SSS) method to investigate whether there is some kind of dynamics in irregular fluctuations, even if they are modulated by long-term trends or periodicities [7]. However, the SSS method cannot indicate whether data are linear or nonlinear because both linear and nonlinear data have some kind of dynamics and are therefore consistent with the alternative hypothesis. In this paper, we introduce a method to investigate whether there is nonlinearity in irregular fluctuations (short-term variability) even if they exhibit long-term trends. The method is an intuitive modification of previously proposed linear surrogate methods.

The proposed method is composed of two premises: (i) frequencies of irregular fluctuations are higher than those of long-term trends and (ii) when data are linear, if the power spectrum is preserved even if all of the phases are different, we can treat such data as linear data from the same population. This superposition principle is valid only for linear data and not for nonlinear data. We focus our attention on these points and propose a method using this idea. The purpose of our method is to investigate nonlinearity in irregular fluctuations, even if they have long-term trends.

After describing our technique, we will present our choice of discriminating statistic. Then, we will apply the algorithm to two cases using simulated time-series data: (i) data with no trend (this case can also be adequately addressed with the previously proposed linear surrogate methods); (ii) data having long-term trends (this case is not consistent with the linear surrogate methods). In each case, the data we use are both noise-free and contaminated by 10% Gaussian observational noise. Based on the results, we apply the method to several actual time series: nuclear magnetic resonance (NMR) laser data, monthly global average temperature (MGAT), and monthly sunspot numbers (MSN).

II. CURRENT TECHNOLOGIES

The primary contribution of this paper is an alternative method of producing surrogate data sets by which we can investigate nonlinearity in irregular fluctuations even if they exhibit long-term trends. The previously proposed linear surrogate methods (FT, AAFT and IAAFT) are designed to generate flawless linear data [1,2]. The basic strategy of these methods is as follows. One first applies the Fourier transform to the original data, randomizes the phases, and then inverts the transform using the randomized phases. In these methods, all phases are randomized so as to eliminate any nonlinearity in data and then the data can be treated as completely linear [23].

Further concerns have been raised over the application of the AAFT surrogate for almost periodic data (data with strong periodic components) [8]. It might be possible to describe periodic behavior by a special case of the linear autoregressive moving average (ARMA) process. However, time series exhibiting such regular persistent fluctuations are inconsistent with linear noise [9].

Hence, although the linear surrogate methods are effective for irregular fluctuations, the methods are not effective for

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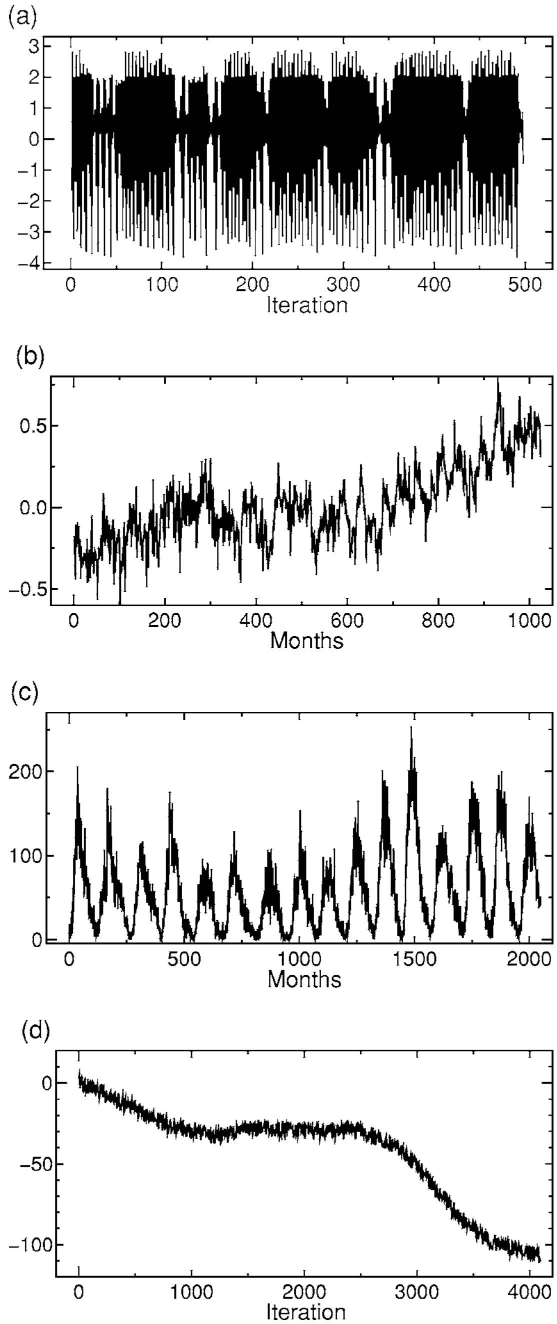

FIG. 1. Segments of four time series examined in this paper: (a) nuclear magnetic resonance (NMR) laser data, (b) monthly global average temperature (MGAT) from September 1920 to December 2005, (c) monthly sunspot numbers (MSN) from January 1749 to 10 August 2004, and (d) x component of the Ikeda map data with artificial trends.

data that exhibit long-term trends because the methods cannot preserve such trends.

III. A DIFFERENT ALGORITHM

It is important that surrogate data are sufficiently similar to the original. That is, when data have long-term trends, it is preferable to preserve those trends. To investigate nonlinearity in irregular fluctuations (especially when they are modulated by long-term trends or periodicities), we want to destroy nonlinearity in irregular fluctuations and preserve the global behaviors (trends or periodicities). When data exhibit irregular fluctuations and long-term trends, the power spectrum is usually like Fig. 2. In particular, the data in Figs. 1(b) and 1(c) have similar spectra to that of Fig. 2 (see Sec. VII). Figure 2 indicates that the data have large peaks of power in the lower-frequency domain and power in the higher-frequency domain is almost white. From this figure we conclude that the higher-frequency domain is probably dominated by irregular fluctuations. This implies that even if we randomize phases in the higher-frequency domain f_ε (see Fig. 2), the influence for long-term trends will not be significant. Hence, to generate data that can fulfill such conflicting conditions (destroying local structures or correlations in short-term variability and preserving long-term behaviors), we randomize phases only in the higher-frequency domain and do not alter low-frequency phases. In this way, long-term trends are preserved in these unaltered low frequencies. This approach is in contrast to previously proposed linear surrogate methods, where all phases are randomized. We call our method the "truncated Fourier transform surrogate (TFTS) method." Since some phases are untouched, TFTS data may still have nonlinearity. However, it is possible to discriminate between linear and nonlinear data. In our approach, we take advantage of different features of linear and nonlinear data [10]. This is possible because the superposition principle is valid only for linear data. That is to say, when data are nonlinear, even if the power spectrum is preserved completely, the inversed Fourier transform data using randomized phases will exhibit a different dynamical behavior. Hence, the null hypothesis addressed by our algorithm is that irregular fluctuations are generated by a stationary linear system. We note that a similar idea (that some phases are untouched to preserve periodicities) is described in [11], however the author

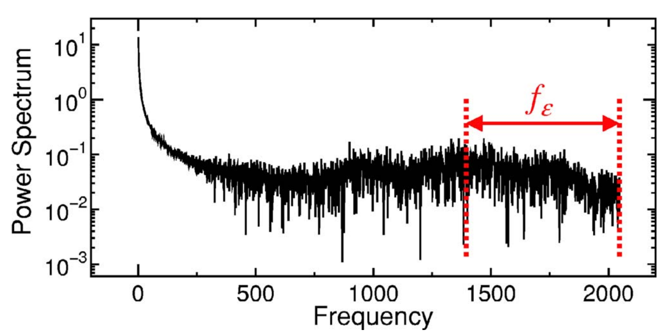

FIG. 2. (Color online) The estimated power spectrum of the artificial data shown in Fig. 1(d), where we use 4096 data points. Note the logarithmic scale. We randomize phases in the higher-frequency domain f_ε and other phases are untouched. The parameter f_ε is the ratio of high-frequency domain to the whole frequency domain. For example, when phases with frequency between 1500 and 2000 are randomized (that is, 500 higher-frequency domain), f_ε is 500/2000, that is, f_ε=0.25. We note that when showing a power spectrum, these usually correspond to each frequency with units of hertz (Hz) on the horizontal axis. In this paper, to explain our proposed method more easily we use an arbitrary scale that corresponds with the number of data points.

did not intend to investigate nonlinearity in irregular fluctuations with long-term trends.

Frequency domain to randomize phases

Obviously, the surrogate data generated by our method are influenced primarily by the choice of frequency domain f_ε [24] (Fig. 2). If the domain is too narrow, the randomization of phases is very little or not at all, and then the TFTS data are almost identical to the original data. In this case, even if there is nonlinearity in irregular fluctuations, we may fail to detect nonlinearity. Conversely, if the domain is too wide, the number of randomized phases is large, and the TFTS data are almost the same as the previously proposed linear surrogate data and the long-term trends are not preserved. In this case, even if there is no nonlinearity in irregular fluctuations, we may wrongly judge that there is nonlinearity in irregular fluctuations. That is, for data with trends, larger values are not appropriate, because the global behavior of the original data is lost. Hence, the smaller the domain the better, provided the domain can destroy local structures and preserve the long-term behavior.

However, we usually cannot determine an adequate value for f_ε *a priori*. It clearly depends on the nature of the data and the length of the time series. Hence, we increase f_ε to randomize the phases from higher domain to lower domain step by step, for example by every 0.05 or 0.1. We continue until linearity and long-term trends are preserved in the surrogate data. We describe the stopping criterion in detail in Sec. V C.

It should be noted that there is a possibility that only when all phases are randomized can nonlinearity in irregular fluctuations be detected. However, our proposed method does not randomize all phases as mentioned above. Hence, our algorithm clearly fails to detect the nonlinearity for such data. Hence, even if the results obtained by applying our method indicate that our null hypothesis is not rejected (that is, nonlinearity in irregular fluctuations cannot be detected), we still cannot get past the possibility that the irregular fluctuations include nonlinearity. However, if the null hypothesis is rejected, it will be strong evidence that there is some kind of nonlinearity in the irregular fluctuations.

IV. THE FOURIER TRANSFORM PROBLEM

There is a problem with linear surrogate algorithms using the Fourier transform. This problem is related to the Fourier transform rather than linear surrogate algorithms themselves. Any implementation of the discrete Fourier transform assumes that the time series under consideration is periodic with some finite period. However, this is not always the case. When there is a large difference between the first and last points as in Figs. 1(b)–1(d), the Fourier transform will treat this as a sudden discontinuity in the time series. As a result, this will introduce significant spurious high-frequency power into the power spectrum—a critical problem when the randomization is centered only on the high-frequency part. This so-called wraparound effect introduces significant bias in the estimated linear properties of the power spectrum [12]. Thus, if we use surrogate data generated in this way, we may wrongly judge the existence of nonlinearity in irregular fluctuations. In particular, when data exhibit long-term trends, the end-point mismatch is rather common and the problem is considerable.

To ameliorate this artifact, when we calculate the power spectrum of such data, we *symmetrize* the original data first. By this procedure, there is no end-point mismatch in the data. The Fourier transform process is then not critically affected by the wraparound effect. Other operations are the same as the TFTS method. Hence, we call the method the symmetrized TFTS (STFTS) method (for more details of the wraparound effect, see, e.g., [9,11,12]). We apply the TFTS method to data with no trend or data with long-term trends when there is no strong end-points mismatch, and STFTS method to data with long-term trends when there is an end-points mismatch.

We note that we use the IAAFT algorithm to apply our idea in this paper, however it is possible to use the FT and AAFT algorithms directly. The reason why we use the IAAFT method is that IAAFT surrogate data have the same probability distribution (rank distribution) as the original data, whereas FT surrogate data do not, although the power spectrum of the FT surrogate is the same as that of the original data. Also, IAAFT surrogate data have much lower deviation of its power spectrum from the original. More details concerning linear surrogate methods and the relevant problems may be found elsewhere [4,9,11–13].

V. HOW TO REJECT A NULL HYPOTHESIS

Discriminating statistics are necessary for hypothesis testing. After calculation of the statistics, we need to inspect whether the null hypothesis shall be rejected. Also, we need to inspect whether linearity and long-term trends are preserved in surrogate data.

A. The discriminating statistics

Dynamical measures are often used as discriminating statistics. The correlation dimension [14] or a Lyapunov exponent [15] are popular choices. To estimate these, we first need to reconstruct the underlying attractor. For this purpose, a time-delay embedding reconstruction is usually applied [16]. The method is most useful when the data exhibit only one characteristic time scale [17]. The method is less effective for data exhibiting irregular fluctuations and long-term trends. This is because a smaller time delay is necessary to treat irregular fluctuations and a larger time delay is necessary to treat long-term trends. At the moment, there is no good method to embed such data.

We choose to use the average mutual information (AMI) as a discriminating statistic [15]. AMI is a nonlinear version of autocorrelation (AC) on a time series. It can answer the following question: On average, how much does one learn about the future from the past? We consider different data realizations from the same population to have the same information flow so that the behavior of the AMI is the same. Furthermore, as we do not need to embed data to estimate

AMI, we can avoid the difficulties associated with embedding.

It is widely observed that estimating AMI is difficult [18]. The major reason is that it is not easy to estimate the underlying probability distribution reliably. To reduce this problem, a new method, using an adaptive partition, has been proposed [19]. However, our surrogate data have the same probability distribution (rank distribution) as the original data. In this case, we consider that the influence due to using different data (the original data and the surrogate data) for estimating the joint probability distribution is not large, and we find that there is no significant bias between the estimated joint probability distribution of the original and surrogate data. Hence, we expect that it is relatively straightforward to compare the AMI of the original data and the surrogate data.

B. Monte Carlo hypothesis testing

After calculation of the test statistic for both data and surrogates, we need to inspect whether a null hypothesis shall be rejected or not. If there is a sufficient difference between the original and surrogate data, the null hypothesis is rejected. In this case, we consider that the original and the surrogate data did not come from the same population. If there is no significant difference, one may not reject the null hypothesis. In this case, we consider that the original and the surrogate data may come from the same population. We employ Monte Carlo hypothesis testing and check whether an estimated statistic of the original data falls within or outside the distribution of the surrogate data [20]. When the statistics fall within the distribution of the surrogate data, the null hypothesis may not be rejected. We generate 99 surrogate data and hence the significance level is 0.01 for a one-sided test.

It should be noted that although the multiple comparison problem is common in surrogate data applications, we show plots of the AMI as a function of time lag (in other words, the variation of the AMI with lag is shown). However, in all cases the hypothesis testing is robustly conducted for lag 1 only. In fact, we expect that it is only a meaningful test statistic for small lag, because the AMI of the original and surrogate data will coincide for large lag. The plots of the AMI as a function of lag are provided for information only.

C. Stopping criterion for increasing frequency domain

We usually cannot know the adequate size of the frequency domain f_ε over which to randomize phases. It clearly depends on the nature of the data and the length of the time series. To know whether linearity and long-term trends in the original data are preserved in surrogate data, it is important to know the size of f_ε, although this is not necessary when data have no long-term trend. To know a rough upper bound of f_ε, we estimate the power spectrum of the original data. Then we determine the frequency domain where the power spectrum is almost white. This gives a good indication of the rough upper bound of the frequency domain to randomize [25].

Also, as mentioned above, we increase f_ε to randomize the phases from higher domain to lower domain step by step. As f_ε increases, we need to inspect whether linearity and long-term trends are preserved in the surrogate data, although this is not necessary when the data have no long-term trend. In addition to visual inspection, we check the AC of the original data and the surrogate data. If we have infinite data, linearity and long-term trends are preserved, and the AC of the original data is identical to that of surrogate data and falls within the distribution of surrogate data as well. However, this is not the case in practice. Convergence to the same power spectrum is not actually guaranteed even under the IAAFT method for a finite number of data points. However, we observe that even if estimates of the power spectra are not identical, the global behavior of the AC of surrogate data is almost identical (or very similar) to that of the original data [9]. We find that especially when data have long-term trends, this is more common. We inspect the AC at time lag 1 because the AC at time lag 1 must be most sensitive to the nature of the data. Hence, we inspect whether the AC of the original data at time lag 1 falls within or outside the distribution of surrogate data. When the AC falls within the distribution, we consider that linearity and long-term trends are sufficiently preserved in the surrogate data, and then calculate the AMI. When the AC falls outside the distribution, we consider that linearity and long-term trends are not well preserved in the surrogate data. Then, we do not use the data, stop increasing f_ε, and adopt the last result (this is the case of using the widest f_ε in the successful application). We note here that even if the AC of the original data at time lag 1 falls outside the distribution of surrogate data, the global behavior of the AC is usually similar.

VI. NUMERICAL EXAMPLES

We now demonstrate the application of our algorithm and confirm our theoretical arguments with several cases. As irregular fluctuations we consider a linear autoregressive (AR) model and the Ikeda map. In all cases, the number of data points used is 4096, and the data are both noise-free and contaminated by 10% Gaussian observational noise.

A. Irregular fluctuations with no trend

The first application is to data with no trend. Irregular fluctuations are generated by following two models:

(i) The linear AR model given by $x_t = a_1 x_{t-1} + a_6 x_{t-6} + \eta_t$ [21], where we use $a_1 = 0.3$, $a_6 = 0.2$, and η_t is Gaussian dynamical noise with standard deviation 1.0.

(ii) The Ikeda map given by

$$f(x,y) = (1 + \mu(x\cos\theta - y\sin\theta), \mu(x\sin\theta + y\cos\theta)),$$

where $\theta = a - b/(1+x^2+y^2)$ with $\mu = 0.83$, $a = 0.4$, and $b = 6.0$ [22].

In each case we use x_t as the observational data. When the data have no trend, this application of the TFTS method is essentially the same as that of the original IAAFT method. Hence, we do not describe any result of the linear AR model because the result obtained by applying the TFTS method to linear data is obvious. However, to demonstrate our premise that the superposition principle is not valid for nonlinear data, we show the result of the Ikeda map under a rather difficult case, where data are contaminated by 10% observational noise and $f_\varepsilon=0.01$ (that is, phases only in 1% frequency domain are randomized) [26]. The AMI of the Ikeda map data falls outside the distributions of the surrogate data, which indicates that the data are not linear [27]. We note that some differences clearly appear when the time lag is relatively small because phases in only the high-frequency domain are randomized and the information in the system is not retained for longer periods of time. In all cases, we can discriminate all data correctly.

B. Irregular fluctuations with trends

The second application is to data with trends. Data generated using the same models as above are added to the artificial trends, and the level of additional data to the trends is equivalent to 10% (20 dB) observational noise at each case. See the behaviors in Fig. 1(d). Also, we use random-walk (RW) data. The model of a RW can be given by

$$x(t+1) = x(t) + \eta(t),$$

where $\eta(t)$ is Gaussian random numbers with standard deviation 1.0. It is well known that as RW will exceed any bounds after finite time, they are treated as *nonstationary*, although the structure of the formula remains unchanged [11]. Hence, linear surrogate methods do not work for RW data. We apply the STFTS method to these data as an *extreme* case to demonstrate our premise.

When irregular fluctuations are the linear AR model data, the data are noise-free and $f_\varepsilon=0.99$ (that is, only 1% frequency domain is untouched) [28], the behavior of the AC of the original data and the surrogate data is very similar, and the AC of the original data at time lag 1 falls within the distribution of the surrogate data like that shown in Figs. 7(d) and 7(e). This fact indicates that linearity and long-term trends in the original data are preserved in the surrogate data, even when $f_\varepsilon=0.99$. The AMI of the original data falls within the distribution of the surrogate data like that shown in Fig. 7(f). Hence, we conclude that we cannot detect nonlinearity in irregular fluctuations: this is the correct result. When we randomize over a smaller f_ε, the results are essentially the same. Also, when we use RW data, the results indicate that the irregular fluctuations are linear, where the rough upper bound of f_ε seems to be 0.8 and we obtain the consistent results up to $f_\varepsilon=0.9$.

With irregular fluctuations from the Ikeda map data, noise-free data, and $f_\varepsilon=0.01$, the AC of the original data is almost identical to that of the surrogate data and the AC of the original data at time lag 1 falls within the distribution of the surrogate data. This result indicates that linearity and long-term trends in the original data are adequately preserved in the surrogate data. However, even when $f_\varepsilon=0.01$, the AMI of the original data falls outside the distribution of the surrogate data [like that shown in Fig. 7(c)]. Hence, we consider that the irregular fluctuations include nonlinearity, which is the correct result. When the time lag is larger, the behavior of the AMI of the surrogate data is very similar to that of the original data. This indicates that the local structures are destroyed and the global structures are preserved in the surrogate data. When data are contaminated by 10% observational noise, we can still detect linearity and nonlinearity in irregular fluctuations correctly by applying our method.

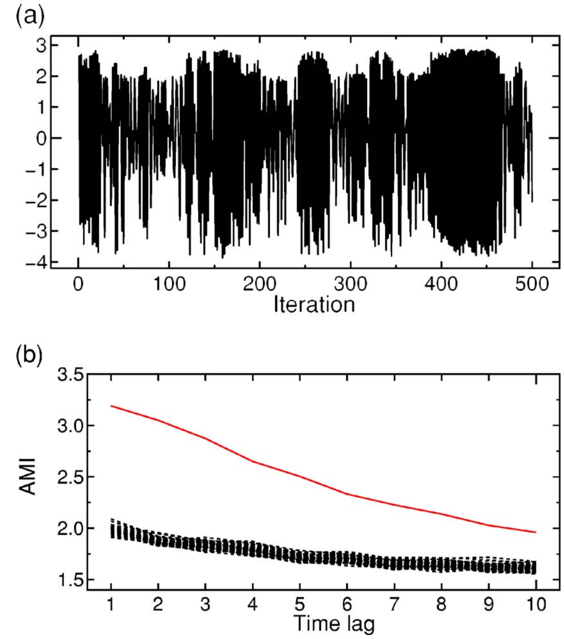

FIG. 3. (Color online) (a) Segments of the surrogate data and (b) a plot of AMI for the NMR laser data: We use $f_\varepsilon=0.1$ and 99 surrogate data. The solid line is the original data and dotted lines are the surrogate data.

The above results show that when irregular fluctuations are linear, the AMI of the original data falls within the distributions of the surrogate data. When irregular fluctuations are nonlinear, the AMI is distinct and falls outside of the distributions. Therefore, applying the TFTS and STFTS methods can detect whether irregular fluctuations are linear or nonlinear using AMI.

VII. APPLICATIONS

Based on the result of these computational studies, we apply the proposed method to three experimental systems: (i) nuclear magnetic resonance (NMR) laser data, which have been known to be nonlinear [14]; (ii) monthly global average temperature (MGAT) from September 1920 to December 2005; and (iii) monthly sunspot numbers (MSN) from January 1749 to 10 August 2004. The MGAT and MSN data seem to have trends. See Figs. 1(a)–1(c), respectively. We use 2048 data points for the NMR laser data and the MSN data, and 1024 data points for the MGAT data. We apply the TFTS method to the NMR laser data because the data have no trend and the symmetrized TFTS (STFTS) method to the MGAT and MSN data because the data have long-term trends and there is the end-point mismatch.

Figure 3 shows a segment of the surrogate data of the NMR laser data and the result, where we use $f_\varepsilon=0.1$ (that is, we randomize phases in the highest 10% frequency domain). Figure 3(a) shows some difference between the original data

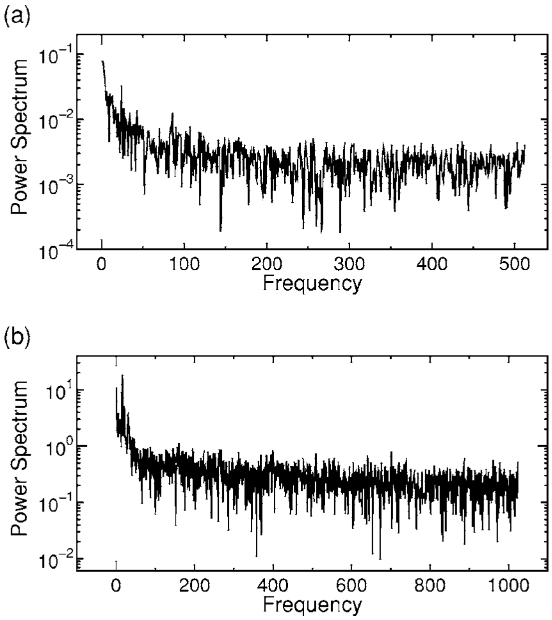

FIG. 4. The estimated power spectrum of (a) MGAT data and (b) MSN data.

and the surrogate data, and Fig. 3(b) shows that the AMI of the NMR laser data fall outside the distributions of the surrogate data. Hence, we consider that the NMR laser data are nonlinear. This result is in agreement with the previously obtained understanding of the data [14]. We note that although we do not randomize all phases like the IAAFT method, we still obtain consistent results.

We now apply the STFTS method to the MGAT and MSN data. We increment f_ε in steps of 0.05. To know a rough upper bound of f_ε, we first estimate the power spectrum of the data. Figure 4 shows that the power seems to be white when the frequency is larger than around 100 for both data sets. The result indicates that the rough upper bound of f_ε is 0.8 for the MGAT data and 0.9 for the MSN data. Hence, we may increase f_ε at most up to the upper bounds, if the AC of the original data at time lag 1 falls within the distribution of surrogate data. When we apply the method, we find that the AC of the MGAT and the MSN data at time lag 1 falls outside the distribution of the surrogate data when $f_\varepsilon=0.6$ and 0.3, respectively. These values are smaller than the upper bounds for both the data.

TABLE I. Frequency domain when the null hypothesis is rejected and not rejected. The R indicates that the null hypothesis is rejected and the NR indicates not rejected. Hence, R implies that our method detects some kind of nonlinearity in the irregular fluctuations, and NR implies that our method fails to detect it. In all results shown in this table, the AC of the original data at time lag 1 falls within the distribution of surrogate data.

	Frequency domain f_ε	
	NR	R
MGAT data	0.05–0.3	0.35–0.55
MSN data	0.05–0.25	

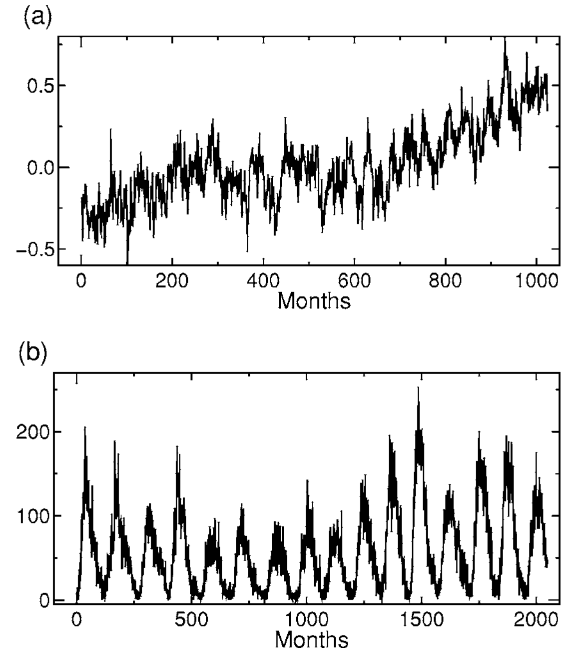

FIG. 5. Surrogate data of time series shown in Figs. 1(b) and 1(c). (a) MGAT data and (b) MSN data.

We show all results in Table I when the AC at time lag 1 falls within the distribution. When the data are the MGAT time series, the null hypothesis is not rejected between 0.05 and 0.3 of f_ε, and the null hypothesis is rejected between 0.35 and 0.55 continuously. When the data are the MSN time series, the null hypothesis is not rejected between 0.05 and 0.25. As mentioned previously, we adopt the last result.

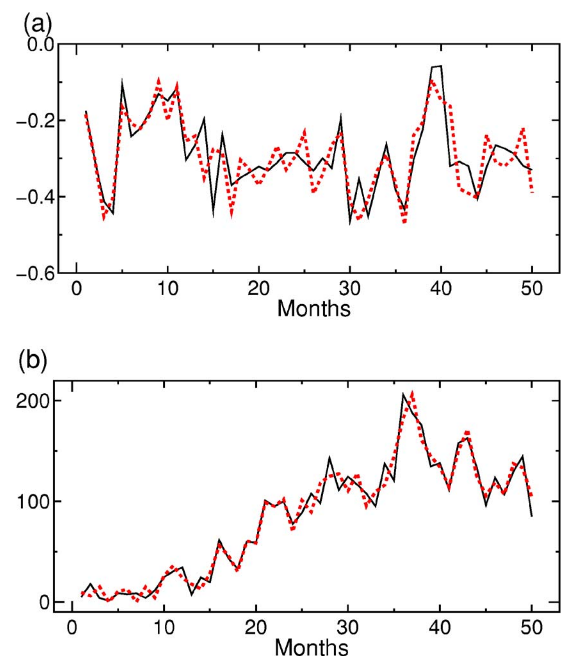

FIG. 6. (Color online) An enlargement of the original data and one of the surrogate data. (a) MGAT data and (b) MSN data, where the solid line is the original data and the dotted line is the surrogate data.

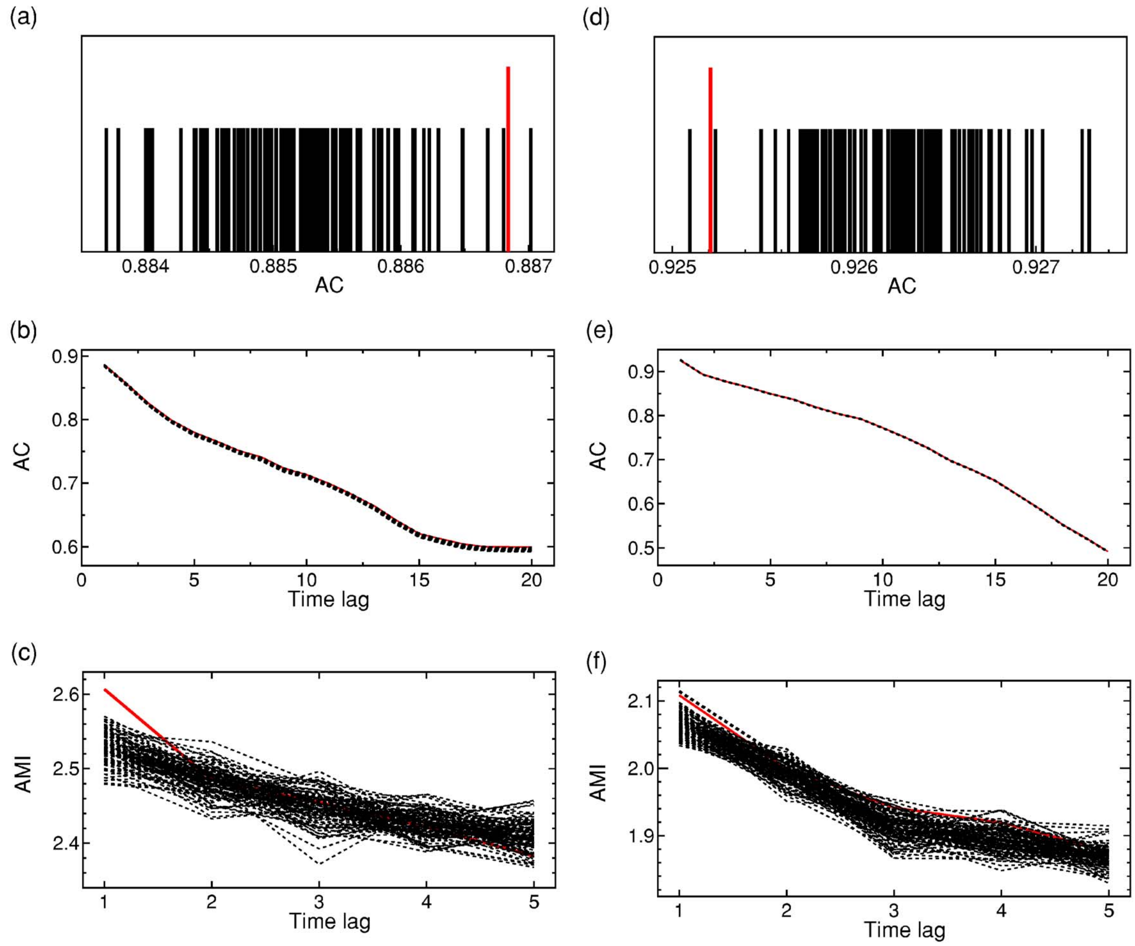

FIG. 7. (Color online) A plot of the AC and the AMI: (a)–(c) MGAT data and (d)–(f) MSN data. In (a) and (d), the longer and short lines correspond to the AC at time lag 1 of the original data and the surrogate data, respectively. In (b), (c), (e), and (f), the solid line is the original data and the dotted lines are the surrogate data.

Hence, we show results for the MGAT data when $f_\varepsilon=0.55$ and for the MSN data when $f_\varepsilon=0.25$. Figure 5 shows the surrogate data for the MGAT and MSN data. Figures 5(a) and 5(b) show very similar behavior to Figs. 1(b) and 1(c), and this indicates that the global behavior is preserved in the surrogate data. However, as Fig. 6 shows, local structures are different between the two. Figure 7 shows the AC and the AMI of the MGAT data, the MSN data, and the surrogate data. Figures 7(a) and 7(d) show that the AC of the original data falls within the distribution of the surrogate data in both cases. Figures 7(b) and 7(e) show that the AC of the original data is almost identical to the surrogate data. From these figures we conclude that linearity and long-term trends are preserved in the surrogate data. Figure 7(c) shows that the AMI of the MGAT data falls outside the distribution of the surrogate data, and Fig. 7(f) shows that the AMI of the MSN data falls within the distribution of the surrogate data. Hence, we consider that we can detect nonlinearity in irregular fluctuations of the MGAT data and we cannot detect that of the MSN data.

VIII. CONCLUSION

By taking advantage of different features of linear and nonlinear data, we described an algorithm to provide data sets for testing nonlinearity in irregular fluctuations. The major difference from previously proposed linear surrogate methods is that we do not randomize all phases but randomize phases of the higher-frequency domain only. This method can be applied to data even if they exhibit long-term trends. Our arguments and computational examples show that this algorithm succeeds in testing nonlinearity and discriminating well between linear and nonlinear data.

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[23] AAFT and IAAFT surrogate data are obtained by rank-ordering of the original data. Hence, in the strictest sense, these surrogate data may still include some kind of *static* non-linearity.
[24] When phases in 0.1 higher-frequency domain are randomized (that is, $f_\varepsilon=0.1$), it means, in other words, that other phases of 0.9 frequency domain are untouched.
[25] For example, Fig. 2 shows that the power spectrum curves from 0 to 400 and the power spectrum at another domain seem to be white. Hence, we expect that we can randomize phases between 400 and 2000 at most, which corresponds to $f_\varepsilon=0.8$.
[26] The smaller the frequency domain to randomize, the more the surrogate data are similar to the original data. This situation is difficult.
[27] The behavior of AMI against time lag is similar to that shown in Fig. 7(c).
[28] Figure 2 shows that the power spectrum is almost white when frequency is between 400 and 2000. Hence, as we mentioned previously, we usually randomize phases up to 400 (that is, $f_\varepsilon=0.8$). We just use this case as an extreme example.